



Later Life in the United Kingdom 2019

This factsheet is a collection of statistics on ageing and later life. It is compiled by Age UK from publicly available sources of research and statistics. This factsheet is not intended to be a fully comprehensive compendium. Where appropriate, we signpost to additional sources of information, statistics and reading.

Wherever possible, this factsheet focuses upon the older population of the UK.

Unless otherwise stated, the term "older" refers to persons aged 65 and above.

Age UK has a number of very informative policy position papers with facts and information on many of the topics covered in this document accessible via the website:

https://www.ageuk.org.uk/our-impact/policy-research/policy-positions/

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- "64' /+#' !6\$) -3(*0!#(20!-3)\$H+(6!(*9!4.(*)#\$*9\$.!5\$/56\$!(.\$!/04\$*!#./+5\$9!4/#\$4' \$.0!4' \$2!(.\$! 931\$.)\$!(*9!' (1\$!1(.3\$9!(*9!93)5(.(4\$!*\$\$9)\tag{4}]?VU<
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- FLRS!5\$/56\$!(.\$!=*/8*!4/!0(7\$!(!*+, -\$.!/0!-(..3\$.)!8'\$*!(77\$))3*#!)\$.137\$)0!4'\$)\$!7(*!3*76+9\$:!93)7.3, 3*(43/*0!3*(55./5.3(4\$!c+\$)43/*)!(*9!7+.3/)342dS'\$)\$!-(..3\$.)!7(*!5.\$1\$*4!0(3.!\$c+(6!4.\$(4, \$*4!3*!'\$(6!'(*9!)/73(6!7(.\$!)\$443*#)!TLPh<%&0!1]?\\d
- FLRS!3*93139+(6!8' /!(.\$!0./, !\$4' *37!, 3*/.342!7/, , +*343\$)!/.!8' /!' (1\$!93)(-36343\$)!7(*!0(7\$! (99343/*(6!3*\$c+(6343\$)!TAJG0!1]?WIV!!
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- P69\$.!FLR!(9+64)!' (1\$!' 3#' \$.!.3)=!/0!93) (-363420!), /=3*#0!(*9!3*7.\$()\$9!(67/' /6!7/*)+, 543/*! 7/, 5(.\$9!4/!/69\$.!)4.(3#' 4!5\$/56\$!T_(**/*0!N' +=6(0!D!h(*9\$.-3640!1]?WU

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- "./+*9!Ba!/0!4' \$!5/5+6(43/*!(#\$9!V0`!3*!G*#6(*9!631\$!834'!)\$1\$.\$!0.(36420!?la!834'!, /9\$.(4\$!0.(3642)!)\$1\$.\$!0.(3642)!?la!834'!, 369!0.(3642)!R["0!1]?YU<
- K.(342!/77+.)!, /.\$!0.\$c+\$*462!3*!8/, \$*!4' (*!, \$*!T?Va!1)!?IaU!TL(6\$!\$4!(60!1]?014!
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- ", /*#!5\$/56\$!6313*#!834' !0. (36420!930037+643\$)!3*!5\$.0/., 3*#!"@F)!(*9!Z"@F)!8\$.\$!.\$5/.4\$9!-2!0Wa>VMaQ!.\$)5\$7431\$620!1)!?Ba>?0a0!.\$)5\$7431\$62!(, /*#!*/*>0. (3613*93139+(6)!TL(6\$!\$4!(64)!1]?0U<

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- %*(99.\$))\$9!0(66!' (d(.9)!3*!4' \$!' /, \$!(.\$!\$)43, (4\$9!4/!7/)4!4' \$!CJN!3*!G*#6(*9!i MBO!, 3663/*!(**+(662! TAJG0!I]?Y7U4!
- S' \$!4/4(6!7/)4!/0!0. (#36342!0. (74+.\$)!4/!4' \$!%&!\$(7' !2\$(.!' ()!-\$\$*!\$)43, (4\$9!(4!i Mdl-3663/*!8' 37' ! 3*76+9\$)!i ?<?!-3663/*!0/.!)/73(6!7(.\$\delta\J35!0. (74+.\$)!(77/+*4!0/.!(./+*9!i I!-3663/*!/0!4' 3)!)+, !TAJG0!I]?Y7U\d!
- J/)534(6!3*5(43\$*4)!3*!G*#6(*9!\$H5\$.3\$*7\$9!(!c+(.4\$.!/0!(!, 3663/*!0(66)!9+.3*#!4'\$!2\$(.!|]?V;?W!T4'(4!3)!, /.\$!4'(*!VY]!5\$.!9(2U!TX/2(6!_/66\$#\$!/0!A'2)373(*)0!!]?WU<!!
- 00]MY!5\$/56\$!(#\$9!V0`!93\$9!0./, !' (13*#!(!0(66!3*!1]?W0!\$c+(43*#!4/!?M!5\$/56\$!\$1\$.2!9(2∢P0!4'\$)\$!10VOW! 8\$.\$!8/, \$*!(*9!10B\?!8\$.\$!, \$*!TPCN0!1]?Y7U∢!
- ", /*#!/69\$.!5\$/56\$@!'35!0.(74+.\$!3)!4'\$!, /)4!7/, , /*!)\$.3/+)!3*E+.2@!.\$()/*!4/!*\$\$9!\$, \$.#\$*72! (*(\$)4'\$)3(;)+.#\$.2@!(*9!7(+)\$!/0!(7739\$*4(6!9\$(4'!TX/2(6!_/66\$#\$!/0!A'2)373(*)@!]]?YU<!!
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- P*62!(./+*9!B?a!/0!/69\$.!(9+64)!.\$#(3*!7/, 56\$4\$62!4' \$3.!5.\$13/+)!(7431343\$)!/0!9(362!6313*#!0/66/83*#!(!'35! 0.(74+.\$|TS(*#!\$4!(60!1]?VUd
- VVa!/0!5\$.)/*)!(#\$9!V0`!8' /!)+)4(3*!(!'35!0.(74+.\$!836!*/4!0+662!.\$4+.*!4/!5.\$>0.(74+.\$!, /-363420!/*62! BVa!.\$4+.*!4/!3*9\$5\$*9\$*7\$!TS(*#!\$4!(60!1]?VUd
- Z4I3)!\$)43, (4\$9!(./+*9!(!c+(.4\$.!/0!'35!0.(74+.\$!5(43\$*4)!(#\$9!VO`!8366!7/*)\$c+\$*462!*\$\$9!6/*#>4\$., ! 7(.\$!TX/2(6!_/66\$#\$!/0!A'2)373(*)@!]?YU<!
- J35!0. (74+.\$)!(.\$!())/73(4\$9!834'!(!4/4(6!7/)4!4/!'\$(64'!(*9!)/73(6!)\$.137\$)!/0!/1\$.!i?!-3663/*!5\$.!2\$(.! TX/2(6!_/66\$#\$!/0!A'2)373(*)0!!]?YU<
- "4!(*2!/*\$!43, \$0!5(43\$*4)!.\$7/1\$.3*#!0./, !'35!0.(74+.\$)!/77+52!/1\$.!B0V]]!'/)534(6!-\$9)!TB0?0\!3*!

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- ?!3*!0!5\$/56\$!(#\$9!W0`!(.\$!6313*#!834'!)3#'4!6/))e!?!3*!1!5\$/56\$!(#\$9!\]`!(.\$!6313*#!834'!)3#'4!6/))!TXCZRQ! I]?YUd!
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- ?!3*!?M!!5\$/56\$!/1\$.!4' \$!(#\$!/0!V0!TWaU!(*9!!?!3*!V!!5\$/56\$!/1\$.!4' \$!(#\$!/0!Y]!' (1\$!9\$, \$*43(IT?WaU! T"6d' \$3, \$.I)!X\$)\$(.7' !%&@I]?\e!"6d' \$3, \$.I)!N/73\$42@I]?\-Ud!
- N2, 54/,)!3*76+9\$!, \$, /.2!6/))@!7/*0+)3/*!(*9!5./-6\$,)!834'!)5\$\$7'!(*9!+*9\$.)4(*93*#!!T"6d'\$3, \$.I)! N/73\$42@!1]?\Ud
- @\$, \$*43(!3)!(!4\$., 3*(6!7/*9343/*!T"6d'\$3, \$.I)!N/73\$420!1]?\\d!
- "'6d' \$3, \$.m)!93)\$()\$!3)!4' \$!, /)4!7/, , /*!425\$!/0!9\$, \$*43(!TVI a!/0!7()\$)U!0/66/8\$9!-2!1()7+6(.! 9\$, \$*43(!T?WaU!(*9!, 3H\$9!9\$, \$*43(!T?]aU!T"6d' \$3, \$.I)!N/73\$420!I]?\-U<!
- @\$, \$*43(!3)!(!=\$2!7(+)\$!/0!93)(-36342!3*!6(4\$.!630\$e!(!, /.\$!7/, , /*!7(+)\$!4'(*!7(*7\$.0!7(.93/1()7+6(.! 93)\$()\$!(*9!)4./=\$!T"6d'\$3, \$.!)!N/73\$420!1]?\-U<!

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- M\a!/0!/69\$.!5\$/56\$!T\$c+31(6\$*4!4/!/1\$.!0!, 3663/*!3*93139+(6)U!)(2!4'\$!4\$6\$13)3/*!/.!5\$4)!(.\$!4'\$3.!, (3*! 0/., !/0!7/, 5(*2!T"#\$!%&0!1]?OU.!
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- Z*!4' \$!%&!(./+*9!(!c+(.4\$.!/0!5\$/56\$!3*!./+43*\$!(*9!, (*+(6!/77+5(43/*)!), /=\$!(*9!(!4\$*4' !/0!5\$/56\$! 3*!, (*(#\$.3(6!(*9!5./0\$))3/*(6!/77+5(43/*)!), /=\$!TPCN0!1]?Y(U<
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- Z*!4'\$!%&C\$!7/*4.3-+4BT 45 0 0 45 0 O0.2(!) 2 (/) -4 0(!)] TJ ET Q q 0.24 0 0 0.242087.2344 639.68 cm

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- MMa!/0!/69\$.!8/.=\$.)!0\$\$6!+*)+55/.4\$9!-2!4' \$3.!\$, 56/2\$.!8' \$*!34!7/, \$)!4/!7(.\$\$.!(, -343/*)!T" 131(0! I]?YUd!
- [\$93(*!' /+.62!5(2!0/.!8/.=\$.)!3*!4' \$3.!0])!3)!i ?B<00!(*9!i ??<\/]!0/.!8/.=\$.)!(#\$9!V]`0!7/, 5(.\$9!4/! i ?M<BW!0/.!8/.=\$.)!3*!4' \$3.!B])!TPCN0!1]?Y9U<

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- Z*!4' \$!*\$H4!1]!2\$(.)0!4' \$!*+, -\$.!/0!3*93139+(6)!834'!7/, 56\$H!7(.\$!*\$\$9)!3)!5./[\$74\$9!4/!3*7.\$()\$!9+\$!4/!
 , /.\$!5\$/56\$!.\$(7' 3*#! (#\$)!YO`! (*9!4' \$)\$!3*93139+(6)!' (13*#!' 3#' \$.!6\$1\$6)!/0!9\$5\$*9\$*720!9\$, \$*43(0!
 (*9!7/, /.-39342!T&3*#)4/*0!_/, ()>J\$..\$.(0!D!](##\$.0!!]?YU<!!
- ")! 4' \$! 5/5+6(43/*! (#\$)! (*9! 5\$/56\$I)! 7(.\$! *\$\$9)! -\$7/, \$!, /.\$! 7/, 56\$H0! 4' \$! *\$\$9! 0/.!)/73(6! 7(.\$!)\$.137\$)!T0/., (6!(*9!3*0/., (6!!3*4\$*)303\$)!T&\$662!D!&\$**20!1]?YUd!
- _(.\$.)! 5./139\$! (! 839\$!.(*#\$!/0!)+55/.4:!.(*#3*#!0./, !5\$.)/*(6!7(.\$0!\$, /43/*(6!)+55/.40!'\$65!834'! 5.(7437(6!4()=)!)+7'!()!)'/553*#0!(*9!.\$, 3*93*#!/.!#313*#!, \$937(43/*!T&\$662!D!&\$**20!1]?YU<
- _(.\$!7(*!-\$!5./139\$9!-2!0(, 3620!0.3\$*9)!/.!*\$3#'-/+.)!834'/+4!5(2, \$*4!T+*5(39!/.!3*0/., (6!7(.\$U!!/.!13(!)\$.137\$)!)+7'!()!7(.\$!'/, \$)!/.!'/, \$!13)34)!T5(39!/.!0/., (6!7(.\$U!T&\$662!D!&\$**20!1]?YU4!!
- B]a!/0!/69\$.!5\$/56\$!3*!G*#6(*9!.\$7\$31\$!*/!'\$65!/.!1\$.2!63446\$!'\$65!834'!4'\$3.!7(.\$!*\$\$9)!!TS.3##6\$@H]?YU<!
- ?4M!, 3663/*!/69\$.!5\$/56\$!9/!*/4!' (1\$!(77\$))!4/!4' \$!7(.\$!(*9!)+55/.4!4' (4!4' \$2!*\$\$9!T"#\$!%&0!1]?\-U<!
- BYa!/0!/69\$.!5\$/56\$!3*!G*#6(*9!.\$7\$31\$!4'\$!'\$65!4'\$2!*\$\$9!0./, !0(, 362!(*9!0.3\$*9)!TS.3##6\$0!1]?YU
- ?!3*!Y!(9+64)!(.\$!7(.\$.)!T(./+*9!V<0!, 3663/*!5\$/56\$U!T_(.\$.)!%&@!|]?0(U<
- R2!1]BW!34!3)!(*43735(4\$9!4'\$!*+, -\$.!/0!7(.\$.)!8366!3*7.\$()\$!4/!\!, 3663/*!T_(.\$.)!%&0!1]?O(U<
- OYa!/0!7(.\$.)!(.\$!0\$, (6\$0!8'36\$!MBa!(.\$!, (6\$!T_(.\$.)!%&0!1]?0(Ud

ļ

0]! 8366!)5\$*9! M<\! 2\$(.)! /0! 4' \$3.!.\$, (3*3*#!630\$! ()! (*! +*5(39!7(.\$.)0! (*9! (4! (#\$! VO0! I < W! 2\$(.)! TPCNQ! I]?W7U<!!

• A\$/56\$!(#\$9!V0!(*9!/1\$.!, (=\$!+5!(!'3#'\$.!5./5/.43/*!/0!7(.\$.)!T?\aU!4'(*!4'\$2!9/!3*!4'\$!5/5+6(43/*!()! (!8'/6\$!T?IaU!

B<O I /@\$\\#=&K&P\$; \\'@''\#'/)&

- "!7/, , /*!9\$03*343/*!/0!5/1\$.42!3)!6313*#!3*!(!'/+)\$'/69!834'!(*!3*7/, \$!-\$6/8!V]a!/0!4'\$!, \$93(



X\$(\$%\$)7\$&!'6#&

"743/*!/*!J\$(.3*#!F/))dTI]?\(UdK(74)!(*9!03#+.\$)!p!"743/*!/*!J\$(.3*#!F/))dX\$4.3\$1\$9!K\$-.+(.2!?I@I]?\\0!0./,!'445):;;888<(743/*/*'\$(.3*#6/))</.#<+=;(-/+4

FLRS<590!

•

'445:;;888<*(43/*(6(.7'31\$)<#/1<+=;9/7;/5\$*>#/1\$.*, \$*4>

```
'445):;;888<(#$+=</.#<+=;#6/-(6())$4);(#$>+=;9/7+, $*4);.$5/.4)>(*9>5+-637(43/*);.$5/.4)>(*9>-.3$03*#);(7431$>7/,,+*343$);4'$q$7/*/,37q7/*4.3-+43/*q/0q/69$.q>5$/56$q>+59(4$q>4/q>I]?W<590!jXK<[]?IU<P69$.!5$/56$I)!'/+)3*#:!_'/37$0!c+(6342!/0!630$0!(*9!+*9$.>/77+5(43/*<jXK<[]?VU</p>
```

```
4/#$4' $.>&3*#)>K+*9>[ (.7' >I ]?Vq?<590!
\label{eq:cur_dispersion} $\operatorname{CJR_dII} ]?YVCJR_!CG^!JP[ \ G!NS"SZNSZ_N!XGhZG^dX$4.3$1$9!0./, \ ! \ ... ]
        '445:;;888<*'-7<7/<+=;7, );5+-63)';7/*)+, $:;[$93(> $*4.$;@/8*6/(9);u?>N4(4)>I]?Y<590!
CJNdTI]?VUdJ35!0.(74+.$!>!CJNdX$4.3$1$9!K$-.+(.2!??0!1]?<math>VUdJ./, !
        '445);;;888<*')<+=;7/*9343/*);'35>0.(74+.$;!
CJNdTI]?WkZ*937(4/.K(74)dX$4.3$1$9!0./,!
        '445):;;888<*')<+=;N7/.$7(.9;A(#$);1*937(4/.K(74)<()5Hs[$4.3729tY?B0!
 CJNdTI ]?YUdA' 2)37 (6! (7431342!#+39$63*$)!0/.!/69$.!(9+64)!>!CJNdX$4.3$1$9!K$-.+(.2!??0!I ]?\\   0!0./, ! = (1.21??0!I)?  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  0!0.//  
        '445):::888<*')<+=:631$>8$66;$H$.73)$:5'2)37(6>(7431342>#+39$63*$)>/69$.>(9+64):!
CJNdTI]?\(Ud[(6*+4.343/*!]CJNdX$4.3$1$9!"5.36!?0!I]?\00./, !
        '445):;;888<*')<+=;7/*9343/*);, (6*+4.343/*;!
CJNdTI]?\-UdCJN!G*#6(*9{|!"#$3*#!8$66!(*9!)+55/.43*#!5$/56$!6313*#!834'!0.(3642dX$4.3$1$9!
        [ (.7'!|W0!|]?\0!0./, !'445):;;888<$*#6(*9<*')<+=;/+.8/.=;763*37(6>5/6372;/69$.>
        5$/56$;0.(3642;!
CJN!@3#34(&TI]?WU4A-%'(2!E:,;-)!+7,!H19'%1"!KLMT!E70#%'!0%,-!+7,!7'"-,!%": '($!A-%'(2!E:,;-)!+7,!
       H19'%1"!KLMT5!E70#%'!0%, -!+7,!7'"-,!%": '($\d\$4.3\$1\$9!0./, !
        888<)4(43)437)(+4' /.342<#/1<+=;())$)), $*4;7/9$>/0>5.(7437$!
CJN!G*#6(*9<TI]?MI</td>
677'>#(!97(!9-1-,%'!/,%0(#$0-!#1!$: //7,(#19!7'"-,!/-7/'-!'#;#19!8#(2!+,%#'()

       X$4.3$1$9!0./, !' 445):::888<$*#6(*9<*')<+=:85>7/*4$*4:+56/(9):17?W:7B:4//6=34>#$*$.(6>
        5. (7437$>0. (3642<590!
 \texttt{CJN!G*\#6(*94TI]?\ldCJN!G*\#6(*9\{|!G6\$74./*37!K.(3642!Z*9\$H4X\$4.3\$1\$9!\lceil (.7'!IW0!I]?\ld./,!} 
        '445):;;888<$*#6(*9<*')<+=;/+.8/.=;763*37(6>5/6372;/69$.>5$/56$;0.(3642;$03;r8'(4>9/$)>
       4' $>*$8>0. (3642>39$*43037(43/*>.$c+3.$, $*4>3*>4' $>#5>7/*4. (74>, $(*>0/.>#$*$. (6>5. (7437$!
'445):;;888</*)\#/1<+=;7$*)+);I]??7$*)+)!
PCNdTI ]?VIdN$H+(6!39$*43420!%&:!1 ]?VI>!P0037$!0/.!C(43/*(6!N4(43)437)dX$4.3$1$9!K$-.+(.2!?0!1 ]?\0!
       0./, !'445):::888</*)#/1<+=:.$6$()$):)$H+(639$*4342+=I]?V!
PCN < TI ]?W(U < K(, 363$)!(*9!J/+)$'/69)!>!P0037$!0/.!C(43/*(6!N4(43)437) < X$4.3$1$9!K$-.+(.2!??0!)
        1]?\0!0./, !
        '445):;;888</*)<#/1<+=;5$/56$5/5+6(43/*(*97/, , +*342;-3.4')9$(4')(*9, (..3(#$);0(, 363$
       ); -+66$43*); 0(, 363$)(*9'/+)$'/69); I ]?W!
PCNdTI ]?W-UdC (43/* (6!5/5+6(43/*!5./E$743/*):!I ]?V>-()$9!>!P0037$!0/.!C (43/* (6!N4(43)437)d
       X$4.3$1$9!j(*+(.2!B?0!1]?\0!0./, !
        '445):;;888</*)<#/1<+=;.$6$()$);*(43/*(65/5+6(43/*5./E$743/*)1]?V-()$9)4(43)437(6-+66$43*!
PCN4TI ]?W7U4%*5(39!7(.$.)!5./139$!)/73(6!7(.$!8/.4'!i OW!-3663/*!>!P0037$!0/.!C(43/*(6!N4(43)437)4
       X$4.3$1$9!K$-.+(.2!IV0!I]?\0!0./, !
        '445):;;888</*)<#/1<+=;5$\/56$5\/5+6(43\/*(*97\, , +*342;'$(64'(*9)\/73(67(.$;'$(64'(*960)$
        $H5$74(*73$); (.4376$); +*5(397(.$.)5./139$)/73(67(.$8/.4' OW-3663/*; I]?W>]W>?]!
PCNdTI ]?Y(UdC": '(!$<7>#19!2%&#($!#1!(2-!F05!KLMUd' 445):;;9/3</.#;?]<I?]0;"jAJ<\text{WV<??<?BBW!
PCNdTI]?Y-Ud_'(*#3*#!4.$*9)!3*!, /.4(6342!>!P0037$!0/.!C(43/*(6!N4(43)437)dX$4.3$1$9!j(*+(.2!B?0!
       1]?\0!0./, !
        '445):::888</*)<#/1<+=:5$/56$5/5+6(43/*(*97/, , +*342;-3.4')9$(4')(*9, (..3(#$);630$$H5
        $74(*73$); (.4376$); 7' (*#3*#4.$*9)3*, /.4(6342; (7./))+=7/, 5(.3)/*?\Y?4/I]?V!
N4(43)437)0!S(-6$!I < X$4.3$1$9!K$-.+(.2!V0/]?\0!0./, !
```

```
X$4.3$1$9!K$-.+(.2!IW0!I]?\0!0./, !
     '445):;;888</*)<#/1<+=;$, 56/2, $*4(*96(-/+., (.=$4;5$/56$3*8/.=;$(.*3*#)(*98/.=3*#'
    /+.);9(4()$4);(#$#./+5()'$4(-6$V!
PCNdTI]?Y$UdG)43, (4$)!/0!4'$!5/5+6(43/*!0/.!4'$!%&0!G*#6(*9!(*9!^6($)0!N7/46(*9!(*9!C/.4'$.*!
    1.$6(*9!>!P0037$!0/.!C(43/*(6!N4(43)437)<!X$4.3$1$9!K$-.+(.2!00!1]?\0!0./, !
     '445):::888</*)<#/1<+=:5$/56$5/5+6(43/*(*97/, , +*342;5/5+6(43/*(*9, 3#.(43/*;5/5+6(43
    /*$)43, (4$);9(4()$4);5/5+6(43/*$)43, (4$)0/.+=$*#6(*9(*98(6$))7/46(*9(*9*/.4' $.*3.$6(*
    9!
PCN4TI ]?Y0U4G)43, (4$)!/0!4' $!1$.2!/69Q!3*76+93*#!7$*4$*(.3(*)Q!%&!>!P0037$!0/.!C(43/*(6!N4(43)437)4
    X$4.3$1$9!j(*+(.2!B?0!1]?\0!0./, !
     '445):;;888</*)<#/1<+=;5$/56$5/5+6(43/*(*97/, , +*342;-3.4')9$(4')(*9, (..3(#$);(#$3*#
     ; -+66$43*); $)43, (4$)/04' $1$.2/693*76+93*#7$*4$*(.3(*); 1]]14/1]?W!
PCNdTI ]?Y#UdG4' *37342!0(74)!(*9!03#+.$)dX$4.3$1$9!0./, !'445):;;888<$4' *37342\0(74)>
    03\#+.\$)<)\$.137\$<\#/1<+=;-.343)'>5/5+6(43/*;*(43/*(6>(*9>.$#3/*(6>5/5+6(43/*);5/5+6(43/*>/0>
     $*#6(*9>(*9>8(6$);6(4$)4!
PCNdTI ]?Y' UdGH7$))!83*4$.!, /.4(6342!3*!G*#6(*9!(*9!^(6$)!>!P0037$!0/.!C(43/*(6!N4(43)437)d
    X$4.3$1$9!K$-.+(.2!??0!1]?\0!0./, !
     '445):::888</*)<#/1<+=:5$/56$5/5+6(43/*(*97/, , +*342;-3.4')9$(4')(*9, (..3(#$);9$(4')
     ;-+66$43*);$H7$))83*4$., /.4(63423*$*#6(*9(*98(6$);1]?W4/1]?Y5./13)3/*(6(*91]?V4/1]?W03
     * (6!
PCN<TI ]?Y3U</br>
J$ (64' !)4(4$!630$!$H5$74(*73$)0!%&!u [ Z!>!P0037$!0/.!C(43/*(6!N4(43)437)<!X$4.3$1$9!</p>
    K$-.+(.2!?0!1]?\0!0./, !
     '445);;;888</*)<#/1<+=;5$\/56$5\/5+6(43\/*(*97\/, , +*342;'$(64'(*9)\/73(67(.$;'$(64'(*9630$
     $H5$74(*73$);, $4'/9/6/#3$); $(64')4(4$630$$H5$74(*73$)+=c, 3!
PCNdTI ]?YEUdJ/+)$' /69!5./E$743/*)!3*!G*#6(*9!>!P0037$!0/.!C(43/*(6!N4(43)437)dX$4.3$1$9!K$-.+(.2!
    ??0!1]?\0!0./,!
     '445):::888</*)<#/1<+=:5$/56$5/5+6(43/*(*97/, , +*342;5/5+6(43/*(*9, 3#.(43/*;5/5+6(43
    /*5./[$743/*);-+66$43*);I]?V-()$9'/+)$'/695./[$743/*)3*$*#6(*9;I]?V-()$9'/+)$'/695./[
    $743/*)3*$*#6(*9!
PCNdTI ]?Y=UdF313*#!6/*#$.!>!P0037$!0/.!C(43/*(6!N4(43)437)dX$4.3$1$9!j(*+(.2!B?0!I ]?\0!0./, !
     '445):;;888</*)<#/1<+=;5$/56$5/5+6(43/*(*97/, , +*342;-3.4')9$(4')(*9, (..3(#$);(#$3*#
     ; (.4376$); 6313*#6/*#$.'/8/+.5/5+6(43/*3)7'(*#3*#(*98'234, (44$.); I]?Y>]Y>?B!
```

- PCNdTI]?Y6UdC (43/* (6!630\$!4(-6\$)0!%&:!I]?0!4/!I]?W!>!P0037\$!0/.!C (43/* (6!N4(43)437)dX\$4.3\$1\$9! K-.+(.2!?0!1]?\00./, !'445):;888</*) < #/1<+=;.6()$);*(43/*(6630$4(-6$)+=1]?04/1]?W!$
- PCN $\langle TI \rangle$? V $\langle A/5+6(43/*!$)43$, $\langle 4\$\rangle$!-2!, $\langle .34(6!)4(4+)!(*9!6313*#!(..(*#$, $*4)0!G*#6(*9!(*9!^6)!))$ >!P0037\$!0/.!C(43/*(6!N4(43)437)\dX\$4.3\$1\$9!K\$-.+(.2!00!1]?\0!0./, !
 - '445):;;888</*)</ri>'445):;;888*/1<+=;5\$/56\$5/5+6(43/*(*97/, , +*342;5/5+6(43/*(*9, 3#.(43/*;5/5+6(43/*)</td> /*\$)43, (4\$); -+66\$43*); 5/5+6(43/*\$)43, (4\$) -2, (.3-0.4(/)-0.3(5))0.2(0)0.5!(TT71Tf0.1(!)0.2((/)-0.3(,)0...

 $C+.)3^*\#4X\$4.3\$1\$9!K\$-.+(.2!IY0!I]?\0!0./, !' 445):;;888<.7^*</.\#<+=;763^*37(6>4/537);/69\$.>5\$/56\$;, \$^*4(6)' \$(64'>3^*>6(4\$.>630\$!X/2(6!_/66\$\#\$!/0!A' 2)373(*)4II]?WU4$

 $S' $!83*\#)!K+*94TI]?Y(U4N4(663*\#!630$!\$H5$74(*72!3*!4' $!\%8!p!S' $!83*\#I)!K+*94X$4.3$1$9!K$-.+(.2!?0!I)?\\(1)?\\(1)?\\(2!?0!0./, !' 445):;;888<=3*\#)0+*94..\#4+=;5+-637(43/*);)4(663*\#I)$